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TRANSVERSE SYMPLECTIC FOLIATION STRUCTURE FOR DISSIPATIVE THERMODYNAMICS BASED ON SOURIAU MODEL OF STATISTICAL MECHANICS

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ABSTRACT

Jean-Marie Souriau's model, known as "Lie groups Thermodynamics," is a symplectic model of statistical mechanics. This framework integrates geometric methods into statistical mechanics, where Gibbs states of a system are represented as points in a symplectic manifold, and Lie groups describe the symmetries of the system. We give an original interpretation of “Lie groups Thermodynamics” with a symplectic foliation generated by coadjoint orbit of the Lie group acting on the system, where the Entropy is characterized as an invariant Casimir Function. Transverse to this symplectic foliation, considered as level sets of Entropy, we can associate a Riemannian foliation as level set of Energy. These transverse foliations define a webs structure. We explain dynamics along both transverse leaves by a metriplectic flow mixing Poisson bracket on symplectic leaves (level sets of Entropy), describing non-dissipative phenomenon by preserving Entropy, and metric bracket on Riemannian leaves (level sets of Energy), characterizing non-dissipative phenomenon by Entropy production. In the framework of Information Geometry for statistical manifolds, we can associate to the symplectic foliation the Fisher metric, and to the Riemannian foliation the dual of the Fisher metric (hessian of Entropy).

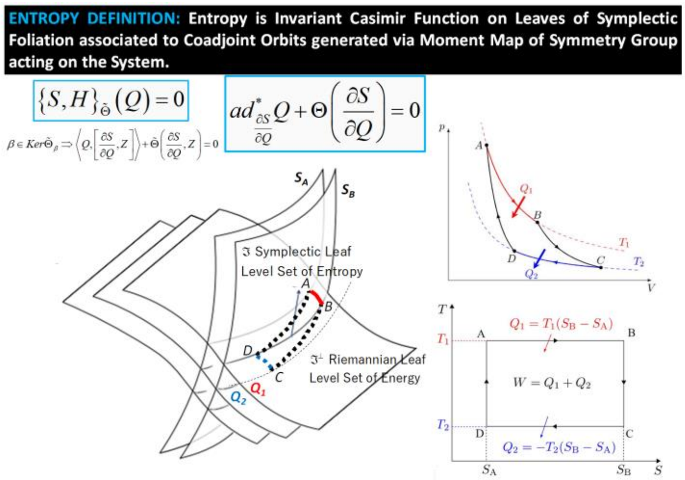

Figure 1: Entropy as Casimir function on Symplectic leaves (level set of entropy) generated by coadjoint orbits and transverse Riemannian leaves (level sets of Energy). Sadi Carnot Cycle on transverse foliations.

We develop this foliation model of thermodynamics, making the link with the notion of metriplectic flow, compatible with Onsager relations, describing the dynamics along each of the leaves. We show that to the symplectic foliation of Souriau, we can associate a transverse Riemannian foliation by the energy level sets, which describe the dissipative phenomena and the generation of Entropy of the 2nd principle of Sadi Carnot. We recall that the physicist Baptiste Coquinot has established the compatibilities of metriplectic flow with the Onsager relations that describe dissipa-tive phenomena. We also recall that Günther Vojta had studied first Onsager rela-tions within the framework of symplectic geometry. We conclude with physicist Herbert B. Callen's insight into thermodynamics as the Science of symmetry. We recall avenues of study opened by Callen to explore more deeply the role of symmetry in thermodynamics.

Keywords: Thermodynamics, 2nd principle, Symplectic Foliation, Riemannian Foliation, Lie Groups, Lie Algebra Cohomology, Entropy, Casimir Function, Pfaff forms

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